Do patents slow down technological progress? Real options in research, patenting, and market introduction

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Abstract

This paper challenges the widely held view in the industrial organization literature that patents always speed up technological progress. We introduce a model of an innovating firm with uncertain property right to its innovation. The impact of a commitment to an R&D project is to create future options for patenting and market introduction. Each decision undertaken will change the conditions in which the innovator operates. It is shown that an increase in patent protection reduces the elasticity of the option value of the program with respect to the value of the project, raising the threshold value of market introduction and enhancing the ability of the innovator to wait. Thus, while the effect of patent is to raise the rents on and thereby the potential amount of innovations, it also tends to slow down market introduction. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

It is common wisdom in the theory of innovation that patents promote technical progress. Our paper challenges this view. We introduce the argument that by
reducing the losses from the rival’s entry, patents also enhance the ability to wait for commercialization and may actually lead to delaying market introduction of new innovation. The reason for such a result is that with patenting the innovator will be less concerned about competitive introductions. Without patenting, an innovation has properties of a public good. We regard the patent as a contract between the innovator and society. By patenting, the patent holder discloses information about its innovation to competitors. Information disclosure has social value in reducing duplication. In exchange, society chooses the scope for protection. Even if lead time may show up as an appealing instrument against competitors, such a hedge basically creates an option value for delaying before commercialization. This phenomenon has not been left unnoticed; van Leuven (1996) says: “In cases where the applicant is not sure about the concrete use of the innovation he may decide to keep his options open for the future and may ask for patent protection.” It has indeed been observed in practice that especially in new markets firms acquire patents and to be able to wait and find out how the uncertain market demand will evolve. This observation points to interpretation of patents as options to commercialize. An informal discussion on this point can be found in Pitkethley (1997).

Such a result will be shown in the following model. First, we regard the investment projects as sequential; each decision undertaken creates a sequence of future options. Second, each decision is viewed as altering the economic conditions, e.g. the market structure, in which the innovator is operating. From the perspective of the real world, it is well-known that there are inactive or ‘sleeping’ patents though such a phenomenon is not very efficiently documented. Firms acquire patents to slow down the progress of potential rivals by forcing the rivals

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1. The authors have consulted Mr Ilkka Rahnasto, a legal counsel for Intellectual Property Rights Management of Nokia Group. He indicated that patents are often left temporarily unused when they arise as distinct from the normal activity of a company, or in other cases when there is considerable uncertainty about future technology or market. Market uncertainty can sometimes be a source of temporary disuse even in actual development projects.

2. The approach of sequential real options has a wide range of applications in other economic or other social contexts. As an example, one can think of dating, leading to an option to propose marriage later which again leads to further options.

3. There is, however, anecdotal evidence. Areeda and Kaplow (1988) discuss the nonuse of patents: “One significant way of exercising a patent is to suppress it—that is, neither to use nor to license it.” They refer to the information in Machlup, An Economic Review of the Patent System 12 (Study No. 15 of the Subcommittee on Patents, Trademarks, and Copyrights of the Senate Committee, 1958) who reports that “It has been estimated that between 80 and 90% of all patents may be in this category.” Furthermore, the Economic Council of Canada (1971) reports that “only 15 per cent of the patents granted in the three years covered by the survey have been worked in this country, while 48 per cent have been worked in other countries.” The report also refers to a US study (by F.M. Scherer) which had found that on average only 54 per cent of patent-holders (covered in a 1956 survey) did in fact put their patent to commercial use. Moreover, the HBS case study (HBS, 1986) points to the practice of the XEROX Corporation of leaving some of its patents unused. Cf. also Footnote 1.
to develop differentiated products. Our approach, however, emphasizes the intrinsic nature of an inactive patent as an option. Some patents will remain in force for some time only and are then left unrenewed. The option to commercialize may not turn out to be economically viable or technological progress elsewhere in the industry may have reduced the value of such an option. For example, Lanjouw (1998) finds that over half of computer patents, whether commercialized or not, are worthless within ten years of their application date. Our model provides a formalization of an incentive to hold inactive patents for some time though with the ultimate aim of commercialization of the new product.

The fascinating study by Long (1991) shows that the historical evolution of the patent institution goes back at least to 13th century Europe. The practice of granting a privilege to an innovator was well-established in the 15th century Venice culminating in ‘the first law of patents’ of 1474. The study of patents by economists came much later, though one should not overlook the fact that Adam Smith’s *The Wealth of Nations* (Smith, 1976) includes remarks on patents. The normative justification for patents in the economic literature, introduced by Machlup (1968) and Nordhaus (1969), emphasized the future welfare gains from new innovations in wiping out the short-run welfare losses resulting from the allocative efficiency loss from monopoly power. Subsequent economic theory has provided further insight into the complex mechanisms associated with the patent institution. Having explored the issue of optimal patent length in his seminal work in 1969, Nordhaus (1972) was also the first to provide an analytical framework to study the impact of patent width. Even though a few examples employing imperfect patent protection existed, e.g. Kamien and Schwarz (1972), his important observation that patent width is significant in preserving incentives to innovate remained forgotten for almost 20 years.

It is possible that it was the 200th anniversary of US patent law, designed to achieve the constitutional objective of promoting the inventive process, that inspired several simultaneous attempts to extend the theory of patents at the beginning of this decade. To mention some of the contributions, Waterson (1990) presents a description of how the coverage of a product patent actually affects market behavior. Klemperer (1990) clarifies the definition of patent breadth in a model of horizontal product differentiation. By assuming that the profits of the innovator are a concave function of the patent width, Gilbert and Shapiro (1990) show that the optimal patent policy prefers narrow but infinitely long patents. Matutes et al. (1996) elaborate the issue of the optimal length and scope of patent

\[4\] An opposite motive cannot be ruled out either. There are products which might have the chance of becoming a standard. This is the case if a sufficiently large market share is captured. In such a case, the innovator might prefer to be imitated by other firms and therefore it might find it attractive to refrain from applying for a patent. Our model focuses on products which are not expected to become product standards.
protection further. They verify the informal argument by Scotchmer (1991) that patent scope is a key consideration in incentives to innovate. Indeed, the main conclusion in their paper is that width of patent protection should be used to induce early introduction of new innovations.

The importance of patent width follows from the fact that acceptance of the patent requires the revelation of information. In Matutes et al. (1996), imitation is possible through either the information included in the patent file or reverse-engineering the products introduced into the market. It is intrinsic to the patent institution that the competitors will have access to information about the innovation. Only exceptionally can patent width be ‘absolute’. Instead, imitation remains an option for the competitors in the form of more costly ‘inventing around’. Below, we emphasize that the incentives of the innovator are determined by patent width and effective patent length, instead of the statutory patent duration. Indeed, as the empirical evidence reported by Levin et al. (1987) suggests, the choice between obtaining a patent and maintaining secrecy may be influenced by the extent to which the disclosures made in the patent document facilitate the imitation. Patents do increase imitation costs, as the query by Mansfield et al. (1981) in addition to Levin et al. (1987) indicates, but they cannot prevent imitation. Gallini (1992) provides an extensive welfare analysis of the patent system in which patent breath increases imitation costs. The fact that patenting involves sunk costs in terms of various agent fees\(^5\) led Pakes (1986) to treat patenting in a novel way as an optimal renewal problem: an inventor faces the problem of whether to pay the annual renewal fee or not.\(^6\)

Besides there being fixed patenting costs, the timing of the patent application is a matter of optimization. There may be a potential trade-off between submitting and hence revealing information early and making imitation more costly, and submitting later thereby increasing the possibility of rival introduction but without

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\(^5\)Patenting is costly, especially within an international context, including, for instance, filing fees, agent fees and translation fees, for discussion see Eaton and Kortum (1994). In the UK, a patent lasts for a maximum of 20 years, during which time annual registration fees must be paid to keep it in force. In 1998, the fee for obtaining a patent stood at a modest £205. The annual renewal fees are based on a sliding scale beginning at £55 in the fifth year of the patent and increasing to £450 for the 20th and final year. The USA, where patents also last for 20 years since 1995 (earlier 17 years), has a complicated fee structure based on a $790 basic filing fee and a $1320 issue fee. It recently also adopted an increasing annual fee to maintain the patent: after 3.5 years $1050, after 7.5 years $2100 and after 11.5 years $3160. If only for simplicity, we lump all patent costs into one variable, \(P\), through discounting the future costs to the date when the patent is approved.

\(^6\)By paying the fee, the patent holder receives the returns from the patented innovation for the next year and the option to maintain the patent over the coming years. As a result, Pakes (1986) was able to estimate the option values of the patent by employing patent renewal data from three European countries.
any helpful knowledge spillovers. A patent gives the innovator time to process information on the trends in market conditions and exogenous uncertainties associated with the demand so as to make ultimately the right decision on the introduction of the new product. Depending on the subsequent expected economic value of the patented idea, the innovator may keep the option of bringing the product onto the market alive for some time. Specifically, knowing that patent protection reduces the losses from the rival’s entry, the innovator might choose to wait longer. Were this the case, patents would actually slow down technological progress. The current paper formalizes this intuition.

We have chosen to model innovation valuation in terms of a continuous-time stochastic process given that innovations are inherently subject to technological uncertainty and given that the expectations of future demand are subject to continuous revisions in light of market changes. Such methods have been widely used in the theory of investment under uncertainty since McDonald and Siegel (1986). The problem of sequential investment decisions under uncertainty has been earlier studied by Dixit and Pindyck (1994). The current paper builds on their methodology with the qualification that commitment of a sunk cost of initiating a research program and the subsequent patenting decision changes the nature of the stochastic process faced by the innovator. This leads to separation over time of optimal timing of commitment to various sunk costs.

Our paper is structured as follows. After presenting the model in Section 2 we begin from the last problem, that is, in Section 3 and Section 4, we deal with the optimal introduction of the new innovation onto the market. Section 5 and Section 6 are devoted to analyzing patent application and launching the research project, respectively. Section 7 concludes the paper with some remarks on testable hypotheses.

2. A model of sequential innovation

Consider a person who has an abstract idea. This person might be Thomas Alva Edison who is wondering whether there could be any market for a light bulb or

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7Such a trade-off, however, is likely to exist only for a short time if information leaks out quickly (cf. Mansfield, 1985; Henderson and Cockburn, 1996). However, firms often rely on secrecy instead of a patent because the impact of the patent is to accelerate the information leakage.

8Our paper differs from Pakes (1986) in two ways. Pakes concentrates on the patent renewal decision while we study the optimal timing of all stages in the innovation process: research, patenting, and market introduction. Moreover, in Pakes’ analysis, decisions undertaken do not change the stochastic processes facing the innovator. We also differ from Matutes et al. (1996) in that they concentrate in sequential applications in the absence of uncertainty.
electricity. Even if there were, the value of this idea would be subject to potential depreciation if his rivals manage to exploit the very same idea\(^9\). The innovator will value the investment opportunity subjectively and initially there might be no sunk costs involved. It is typical, however, that ideas cannot be patented before some non-trivial sunk cost (research investment) is paid.

By committing itself to a research cost, say \(R > 0\), the firm acquires a patent option. However, before the time is right for taking out a patent, the value of the innovation has to be high enough, because patenting is costly. Therefore, we introduce patent cost, \(P > 0\), covering not only the discounted sum of the initial and renewal fees over the expected patent protection period but also the cost of preparing the patent application. Moreover, as emphasized by Waterson (1990), the patent provides the desired protection only if the patent holder stands ready to sue those who infringe the patent. Therefore, it is appropriate to regard \(P\) as including also the potential cost of enforcing patent rights. Both types of costs help to explain the empirical observation that not all innovations are patented.

By reducing the threat of potential competitors, patenting offers the innovator the option of finalizing the project and developing a marketable product. This will require a further investment in the form of a development cost, say \(D > 0\). Our model below is consistent also with the case where patents are acquired for the purpose of selling them out; we work with the background idea that there is a well-functioning market for valuation of rents to an innovation. Such a view becomes important if firms differ in their ability to develop patents into products; an efficient solution would call for separation of innovation and commercialization.

To introduce a formal analysis, assume that in the original stage with no commitment to research, the innovator possesses an investment opportunity. The expected present value of potential rents (profits) on this opportunity, \(V\), is subject to revisions according to a stochastic differential

\[
dV = \mu V \, dt + \sigma V \, dz - V \, dh, \quad V \leq b < \infty
\]

with the following assumptions

**Assumption 1.** The drift is positive, constant and given by \(\mu V > 0\).

We concentrate on those projects which, in the spectrum of ideas, are promising in the sense that the expected rate of evolution of the economic value is positive, \(\mu > 0\). The parameter \(\mu\) represents the evaluation of the innovator about the state

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\(^9\)In fact, this was exactly the situation with Mr Edison. He was aware of George Westinghouse’s rival electrical system. For an amusing story about this competition the reader could have a look at Bryson (1995).
of future markets. Such a view may also arise from expectations of positive network effects enhancing consumers’ willingness to acquire the product\textsuperscript{10}. An example would be the development of mobile phones: an idea about a ‘wireless phone’ has been around since late 19th century, but it took about 100 years to commercialize this idea. Meanwhile, the economic value of the idea has appreciated rapidly. The value of the idea in (1) is subject to three rather different types of uncertainty:

**Assumption 2.** The differential $dz$ represents an increment of a Wiener process with $\sigma^2V^2 > 0$ as its infinitesimal variance; $h$ is a Poisson process with a jump $dh = 0$ with probability $1 - \lambda dt$ and $dh = u \leq 1$, with probability $\lambda dt > 0$; the elimination of the barrier $b$ for the state variable $V$ succeeds with probability $p > 0$.

The economic intuition of the stochastic structure is as follows. Without commitment to a sunk research cost, e.g., without establishing a firm with a research laboratory, the idea can never have economic value. Technically, this view is built in through a ‘barrier’, $V \leq b$ in Eq. (1). Commitment to a sunk cost has a radical implication. It changes the stochastic process (1) to a similar process but with no barrier with the probability $p(R)$ with $p'(R) > 0$, $p''(R) < 0$. It is only the research effort which can provide information about the economic value of the idea. See Section 6 for this analysis. Second, the value $V$ is subject to sudden depreciation due to the risk of exogenous rival patenting. It is assumed that such a risk is realized according to a Poisson process with probability $\lambda dr$. What this means is that the research option may be open for a limited time only. The probability that $dh = 0$, that is, no competitive patent emerges, is therefore $1 - \lambda dr$. Third, the value $V$ depends on the industry-specific risks (stochastic price) and the market structure.

It is useful to think that there is no particular moment when one could say that the ‘breakthrough’ has taken place. Instead, it is useful to think of the evolution of an idea which may or may not turn out to be a commercialized product. Even if a competing innovator proceeds faster resulting in depreciation of the project value of the current innovator, the latter does not need to give up. The roles may, however, be changed. Moreover, what we have in mind in our model is development of completely new product (or new technology) when the innovator faces potential rivals as a general exogenous threat rather than in terms of a well-defined and predictable strategy.

Above we introduced $\lambda dr$, the probability of rival entry. We now show how this probability can be endogenized (though, of course, it remains exogenous for the innovator). We also explore the effects of a patent on this probability below.

\textsuperscript{10}An alternative assumption, $\mu < 0$ would simply point to innovations in declining industries which we will not analyze.
Assume that a potential imitator enters with probability $\lambda$ after the innovator has succeeded. Denote the post-innovation investment of the imitator by $x$. Assuming an exponential distribution, the probability of being successful at a date $t_o$ prior to date $t$ is $\Pr(t_o<t) = 1 - e^{-\lambda(x)t}$, where $\lambda(x)$ is the (twice differentiable) hazard function with $\lambda'(x) > 0$, $\lambda''(x) < 0$ giving the instantaneous conditional probability of success by the imitator as a function of its R&D expenditure $x$.

Such a model of stochastic success has been standard in the theory of innovations in the context of patent race. Note that we are assuming that R&D investment is undertaken at the beginning of the project (cf. the seminal patent race model by Loury (1979)). Earlier models which have assumed that the success probability obeys exponential distribution have regarded the value of innovation as deterministic. In our approach, it is stochastic. It is natural in our context to consider the values of innovation and imitation as determined by the market structure with industry-wide shocks. In our context, the expected discounted profit of the imitator ($F$ for follower) is given by

$$E[\Pi^F] = \max_v \int_0^{\infty} e^{-\lambda(x)+r} \lambda(x) V^F - x \, dr,$$

where $V^F$ is the expected discounted profit of the imitator evaluated at the date of success. Given the properties of the hazard rate $\lambda(x)$, it holds trivially that $d\lambda/dV^F > 0$. Assume then that both firms produce the same commodity and compete in a stochastic market. Assume that shocks are industry-wide, resulting in instantaneous profits $\pi_i = a(P-c)q_i$, $i=1,2$ where the stochastic shift variable $a$ obeys a geometric Brownian motion, $da = a\mu dt + a\sigma dz$. Such shocks are hence assumed to hit both price and costs similarly. Market demand is assumed to be given by $P = \eta - (q_1 + q_2)$; $\eta > 0$. We let $q_i$ denote output per firm, and $c$ stands for the unit cost of production. In the absence of an innovation (and a patent), Cournot competition leads to outputs $q_1 = q_2 = (1/3)(\eta - c)$. This results in instantaneous profits per firm of $\pi_i = aq_i^2$. We now make the assumption that there is actually no market activity before the innovation takes place. This is equivalent to assuming $\eta = c$. The innovation by the innovator makes the production profitable by reducing the cost to the innovator to $c - \theta$ with $\theta > 0$. If successful imitation takes place, the production cost of the imitator is reduced to $c - \theta^F$. The imitator’s cost reduction may be bigger or smaller than that of the innovator. In the latter case with $\theta_p > \theta$, one can speak about an improvement. We now introduce the option that the innovator acquires a patent defined by its width $w \in [0, 1]$ where $w = 1$ corresponds to a perfect patent. As a result, the imitator’s cost of production is $c - (1-w)\theta^F$. Cournot competition again results in profits $\pi_i = aq_i^2$ where this time the distribution of outputs is asymmetric, and is given by $q_1 = (1/3)[2\theta - (1-w)\theta^F]$, $q_2 = (1/3)[2(1-w)\theta^F - \theta]$ (recall that $\eta = c$ by assumption).

For clarity, denote the profits accruing to the innovator and the imitator by $\pi(w)$ and $\tilde{\pi}^F(w)$. The value of successful imitation is given by
Maximization of the expected profit on imitation (2) defined above results in imitation investment $x(w)$ and hence in the hazard rate $\lambda_p(w)$. Trivially, $\lambda_p(w) < 0$.

Although statutory patent length is measured in calendar time, it is the effective duration of the property right to the idea which is economically relevant. Successful inventing-around makes it necessary to re-address the meaning of the effective life of this right. Our model allows us to characterize the effective patent life in terms of the incremental delay to the potential rival’s entry caused by the patent. Without the patent, the expected time for the entry of competitor is given by the inverse of parameter $\lambda$, i.e. $1/\lambda$. The expected duration of monopoly if the firm takes out a patent will be given by $1/\lambda_p$ which could be called the ‘effective’ life of the patent. Notice that it is actually determined by the patent width. The evidence suggests that the expected increase in time of monopoly due to take out of a patent is generally positive but varies across business branches. Exploring 129 industries, Levin et al. (1987) found support for patents diminishing the costs or time required for imitation only in 14 industries.

After market introduction of the product, the innovator faces uncertainty both of demand and the date at which a competitor might enter. We now turn to consider the value of innovation in face of such uncertainties. We illustrate how the patent width influences the value of the project after market introduction. Merely for the sake of simplicity, assume that the statutory duration of the patent is sufficiently long so that the patent will be invented around before it expires. Thus assume that an imitator manages to invent around the patent and introduce a competitive product onto the market at time $s > T$. Recall that the profit flow $\{\pi_t\}$, of the innovator is a submartingale. For the given $s$, the expected present value of the profits at time $T$ is given by

$$V_T(s) = E_T \int_s^\infty e^{-r(t-s)} \pi^F_t(w) \, dt + \int_s^\infty e^{-r(t-s)} \pi^F_t(w) \, dt$$

$$= \frac{\pi^F_T(1)}{r - \mu_u} \left[ \frac{\pi^F_T(1) - \pi^F_T(w)}{r - \mu_u} \right] E_T \left[ e^{-r(s-T)} \right].$$

We note that with $w=1$, the innovator operates as a monopoly. Since the date of competitive market introduction is stochastic ex ante, with $\lambda_p$ standing for the

\footnote{According to Schankerman and Pakes (1986), who evaluated the patent renewal data from Germany, France, and the UK, only 10% of patents are in force for the whole patent life. In other words, $\lambda_p$ is so high that the probability of a successful imitation before the patent expires is 0.9. For similar findings, see also Lanjouw (1998).}
probability that the patent will be invented around during the next time interval, \( dt \), the expected value of the project to the innovator at time \( T \) is given as the discounted value of its short-run monopoly profits \( (w = 1) \) adjusted for reduction in these profits from stochastic entry by the competitor \( (w < 1) \)

\[
V_T(w) = E_T \int_T^\infty (e^{-\lambda_p(s-T)})\lambda_p V_T(s) \, ds
\]

\[
= \left( \frac{1}{r - \mu_u} \right) \left[ \pi_T(1) - \left( \frac{\lambda_p}{\lambda_p + r} \right) (\pi_T(1) - \pi_T(w)) \right].
\]

where, we recall, \( \lambda_p = \lambda_p(w) \). In Eq. (4), \( \exp(-\lambda_p(s-T)) \) is the probability that patent has eluded inventing-around by time \( s \). Clearly, \( V_T(w) \) is increasing in patent width \( w \) and hence decreasing in \( \lambda_p \). Under rational pricing, the value of the patent will be capitalized in the project value immediately even before any research investment has been undertaken. Anyone willing to buy the idea from the innovator would have to pay the capitalized price in the market for ‘ideas’ (should such a market exist). Capitalization also means that the potential buyers could hope to earn only the market rate of return on the patent, given by \( r^{12} \).

If the innovation is not patented (or when \( w = 0 \)), its value is

\[
V'_T(0) = \frac{\pi_T(1)}{r - \mu_u} \left( 1 - \frac{\lambda}{\lambda + r} \right)
\]

if rival entry renders the original innovation unprofitable (the precise treatment of this case is postponed to the end of this section). Thus at time \( T \), the capitalization effect, i.e., the value of the patent can be presented through the difference between (4) and the project value in the absence of a patent:

\[
\Delta = V_T(w) - V'_T(0)
\]

\[
= \frac{1}{r - \mu_u} \left[ \pi_T(1) \left( \frac{\lambda}{\lambda + r} - \frac{\lambda_p}{\lambda_p + r} \right) + \pi_T(w) \frac{\lambda_p}{\lambda_p + r} \right].
\]

Because \( \Delta > 0 \), a patent represents a privilege which raises the value of the investment at the outset. As emphasized by Matutes et al. (1996), the implication is that the threshold value of market introduction may be met earlier when compared to the case of no patent protection. But this is only part of the whole story: by deterring rival entry, a patent also increases the innovator’s ability, not to mention willingness, to wait. We show below that the patent thus has another effect which tends to boost the threshold value by making waiting more valuable.

\[\text{Asymmetric information and moral hazard in the market for ideas would apparently call for compensation for the quality risk.}\]
We can now express the evolution of the value of the project for the patent holder before rival entry as
\[ dV = \mu V \, dt + \sigma V \, dz - V \, dy \]  
(6)
where \( y \) is a Poisson process with a jump \( dy = 0 \) with probability \( 1 - \lambda \, dt \) and \( dy = u_p(w) d\lambda \) with probability \( \lambda \, dt > 0 \).

The wedge \( u - u_p(w) \) depends on patent width. For short-hand notation we denote below \( u - u_p(w) = \gamma(w) \). If only for the sake of simplicity, we explicitly work out below the case with a jump of size \( u = 1 \). This amounts to assuming that the firm will abandon its project if some rival enters before patenting. The reader will notice that such a case arises when imitation leads to an improvement with \( \theta^I > 2\theta \); cf. the expression above for the profit of the innovator with \( w = 0 \). We thus have \( \gamma(0) = 0 \). With perfect patent protection, \( u_p = 0 \), therefore, \( \gamma(1) = 1 \). Thus, \( \gamma'(w) = -u_p'(w) > 0 \).

With favorable trends, the innovator will be ready to commit itself to costly product development. To solve for the sequential investment problem, we rely on Dixit and Pindyck (1994, ch 10). Our results, however, differ from theirs: in our problem, it will not in general be optimal to simultaneously exercise all the three options\(^{13}\) which include: (i) \( F_R \), the option of engaging on a costly research program (with optimal exercise date, say \( t^* \)); (ii) \( F_P \), the option of filing a patent application (with optimal exercise date, say \( T^* \)); and (iii) \( F_D \), the option of undertaking a costly development investment (with optimal exercise date, say \( T^* \)).

3. Market introduction of new innovation

In the last stage what matters for the innovator is the expected present value of the option of commercializing the innovation subject to the choice of decision date such that the value of the project does not allow for riskless arbitrage opportunities. The value of the development option when optimally exercised must then be given by
\[ F_D(V) = \max\{0, \max_T E[(V_T - D) e^{-rT}])\}. \]
(7)
The first max operator allows for the possibility of a permanent ‘sleeping patent’ which never leads to commercialization satisfying \( \max\{0, \max_T E[.]\} = 0 \). Denoting the project value under optimal commercialization by \( V_T^* \), it holds that with\(^{13}\)Dixit and Pindyck proved that in their model with consecutive real options, it is always optimal to pay all the sunk costs at the same time. They also proved that to make it optimal for a firm not to exercise all the options at the same time, one needs to introduce an assumption like an exogenously determined ‘time to build’ capacity. The difference between our results and theirs is due to each decision in our model changing the stochastic environment.
The left-hand side of Eq. (8) expresses the required rate of return on holding the option and the right-hand side provides the expected rate of appreciation in option value per unit of time (in the absence of cash flow before commercialization). Using Ito’s Lemma to expand $dF_D$, using primes to denote the derivatives with respect to the state variable $V$ and recalling $dV$ from Eq. (6), we write the expected capital gain as

$$E[dF_D] = (1/2)\sigma^2 V^2 F''_D(V) \, dt + \mu VF'_D(V) \, dt - \lambda_p [F_D(V) - F_D(\gamma(w)V)] \, dt.$$  

(9)

We recall that $\gamma(w) = 1 - u_p(w)$ and that $F_D(V) - F_D(\gamma(w)V)$ stands for the impact of patent width on the option value of commercializing. It thus turns out that the solution to a second-order differential equation is a good approximation to the expected present value in Eq. (7). Substituting Eq. (9) for Eq. (8) gives such a second-order differential equation in the option value $F_D(V)$

$$(1/2)\sigma^2 V^2 F''_D(V) + \mu VF'_D(V) - (r + \lambda_p)F_D(V) + \lambda_p F_D(\gamma(w)V) = 0.$$  

(10)

Knowing that the innovator optimizes the threshold value $V^*_T$ and that no cash flow is available before the boundary $V^*_T$ has been achieved, the optimal valuation $V < V^*_T$ can include one and only one power term in $V$,

$$F_D(V) = \psi_T V^\alpha.$$  

(11)

In Eq. (11), the coefficient $\psi_T$ is a function which takes a positive value and depends on the threshold. Because it is an upper boundary at which the innovator will make the investment, the exponent of $V$, $\alpha$, cannot be negative. By substitution, we see that Eq. (10) is satisfied by Eq. (11) if $\alpha$ is the positive solution to the nonlinear equation

$$I(\alpha) = (1/2)\sigma^2 (\alpha - 1) + \mu \alpha - (r + \lambda_p)\gamma(w)^\alpha = 0.$$  

(12)

To determine the two unknown functions, $V^*_T$ and $\psi_T$, all arbitrage opportunities must be eliminated. Therefore, we introduce the ‘value-matching’ and ‘smooth-pasting’ conditions

$$F_D(V^*_T) + D = V^*_T$$  

(13a)

$$F'_D(V^*_T) = 1.$$  

(13b)

Stated verbally, Eq. (13a) requires the innovator to pay the sunk cost $D$ and to give up the option $F_D(V^*_T)$ to acquire the expected present value $V^*_T$. Arbitrage opportunities at the margin are eliminated by Eq. (13b). By inserting Eq. (11) into
Eq. (13a) and Eq. (13b) one finds that the threshold and the coefficient are given by
\[ V^*_T = \alpha D/\alpha - 1. \]  
\[ \psi_T = (V^*_T - D)/(V^*_T)^\alpha. \]  

Dixit and Pindyck (1994) have made such results well-known. Intuitively, when the process reaches \( V^*_T \), the firm exercises its option to invest. This takes place at time \( T^* \) which, of course, is stochastic ex ante. As Arnold Plant (1934) put it in his classical study on patents: “A new device, employing recently discovered and revolutionary scientific principle, may be mechanically excellent, and yet not capable of commercial exploitation. The time and extent of its adoption will depend upon price conditions.” Here we consider the question of the effects of patent at the final stage of the project.

4. The effects of a patent on market introduction

The above analysis is fruitful in that it actually has endogenized the optimal lead-time of innovator, often taken to be a more important strategic instrument in competition for markets than patent protection. The current section shows the way in which optimal lead time is affected by access to patent. A patent has two effects in our model. Before the new product exists, a patent affects the optimal innovation strategy and interacts with the options of innovating and commercializing. Moreover, a patent means higher expected profit after market introduction. These effects need to be analyzed separately. We now explore the effects of the patent on the optimal threshold for commercialization.

Eq. (12) is rather involved when \( w > 0 \) (i.e. when \( u < 1 \)), and cannot be explicitly solved for \( \alpha \). It can, however, be established that \( \alpha > 1 \). Totally differentiating Eq. (12), one obtains
\[ \frac{\partial \alpha}{\partial w} = - \frac{\alpha \lambda_p \gamma^{-1} \gamma' + \lambda_p (\gamma'' - 1)}{\partial I/\partial \alpha} \]  
where \( \partial I/\partial \alpha = (1/2) \sigma^2 (2\alpha - 1) + \mu + \lambda_p \gamma(w) \gamma \ln \gamma(w) \). The first two terms in \( \partial I/\partial \alpha \) are positive while the last one is negative. It is, however, easy to show that the positive terms dominate and that \( \partial I/\partial \alpha > 0 \) for all \( w > 0 \). Because the numerator is positive we obtain the result
\[ \partial \alpha/\partial w < 0. \]  

\(^{14}\)The proof follows by finding the minimum for \( \partial I/\partial \alpha \) in terms of \( w \) and observing that the minimum will be positive.
Note that for \( t < T^* \), \( \alpha \) measures the elasticity of the option value with respect to the value of the project (the underlying asset), \( \alpha = \frac{\ln F_{p}(V)}{\ln V} \). We hence report:

**Proposition 1.** A marginal increase in patent width (hence also in effective life of patent) reduces the elasticity of the option value with respect to the value of the project, the underlying asset, raising the threshold value of market introduction.

**Proof.** Rewrite Eq. (14) as \( V^*_T = \alpha(w) D / (\alpha(w) - 1) \). Differentiating the threshold value \( V^*_T \) with respect to \( w \), one immediately finds that \( \partial V^*_T / \partial w > 0 \). QED

Because of the capitalization effect above in Eq. (5), and as explored by the existing research, cf. Matutes et al. (1996), better patent terms raise the value of each project making it possible to achieve any desired threshold earlier. However, our **Proposition 1** implies that there is another and opposite effect. From another perspective, we show that it is possible to solve for the target value of the short-run cash flow which has to be reached before commercialization can be regarded as economically profitable. At the optimal date \( T^* \), it must hold that \( V^*_T(w) \) from Eq. (4) equals the optimal threshold \( V^*_T \) given in Eq. (14). We use this equality to solve for the expected target cash flow as

\[
\pi^*_T(1) = C(w) + B(w) D; \tag{18}
\]

where

\[
C(w) = \frac{\lambda_{p}(w)}{\lambda_{p}(w) + r [\pi^*_T(1) - \pi^*_r(w)]}, \quad B(w) = \frac{\alpha(w)}{\alpha(w) - 1} (r - \mu). \tag{19}
\]

The first term \( C(w) \) captures the capitalization of the cash flow effect. Through this effect, commercialization tends to be speeded up. The contribution of our analysis is to show that there is the option effect, given by \( B(w) \), operating in the opposite direction. Straightforward differentiation of Eq. (18) with respect to patent width and length then yields

**Corollary 1.** Increased patent width delays commercialization if the option effect dominates the cash flow effect, i.e. if \( -dC(w)/dw < dB(w)/dw \).

The case where technological progress is slowed down may seem counterintuitive. The explanation is nevertheless straightforward. Broader patents reduce the impact of a rival introduction on the innovator’s profit. This means a greater option value for waiting when considering market introduction. Intuitively, a patent reduces the threat of potential competition and enhances the possibility to wait and
find out the evolution of market demand: rivals have to try to produce differentiated products not to infringe the patent. In other words, the opportunity cost of waiting is reduced. This result strictly contrasts with that of Matutes et al. (1996), where broader patents unambiguously speed up commercialization. Moreover, it is a standard result in the theory of real options that the value of waiting increases when market uncertainty, measured by $\sigma$, is increased. What this means from the point of view of our problem is that the threshold value given in Eq. (14) will be increased with the implication that delay in market introduction is positively related to the degree of uncertainty. Notice also that the dominance of the option effect is more likely in industries with costly commercialization (large $D$) while the cash flow effect can be expected to dominate in industries with low cost of commercialization. Such a prediction could be subject of empirical testing.

Fig. 1 illustrates the effect of a patent on optimal market introduction in a numerical example. A patent raises the present value of future rents on innovations shifting $V$-curve upwards. At the same time, however, a patent reduces the elasticity of the option value of commercialization relative to the value of the project thereby raising the investment threshold, $V_T^*$. One way to evaluate the result of a rise in the threshold is to claim that without patents, there may be too much and too early investment in innovative activity, as claimed by Barzel (1968). If that is the case, a patent system can be socially valuable in that it tends to reduce competition constraining early commitments in projects with uncertain future prospects.

![Fig. 1. The effect of patent: (i) cash flow effect (the process shifts up); (ii) option effect (the threshold shifts up). The figure plots the sample paths of $V$ for the parameter values $\mu = 0.02$ and $\sigma = 0.4$, with initial values 1 and 4.](image-url)
5. The Hamlet problem: when to be or not to be a patent-holder

A patent is only one instrument in firms’ protection policies and quite often firms choose secrecy instead\textsuperscript{15}. In our approach, we can produce a precise condition for when secrecy is chosen instead of a patent and vice versa. Our model is also consistent with another empirical observation, i.e. that important innovations are likely to be patented.

Before entering the final stage, the firm holds an option to patent its innovation. Since no cash flow will be generated at this stage, and to eliminate riskless arbitrage opportunities, the value of the patent option, \( F_p \), must satisfy the differential equation

\[
(1/2)\sigma^2VF''_p(V) + \mu VF'_p(V) - (r + \lambda)F_p(V) = 0. \tag{19}
\]

The evolution of option value is now different from Eq. (10) because the ‘idea’ is not yet under patent protection and the project value obeys the stochastic differential (1) (with no exogenous barrier), instead of process (6). The boundary conditions which the optimal patent decision has to satisfy are analogous to those above and we can write them as

\[
F_p(V^*_p) + P = F_D(V^*_r) \tag{20a}
\]
\[
F'_p(V^*_p) = F'_D(V^*_r). \tag{20b}
\]

Thus, if the firm decides to patent at time \( t^* \), it will obtain the option of investing in development \( F_D(V^*_r) \), given on the right-hand side of Eq. (20a). The cost of patent has two components, i.e. the lost option value of postponing \( F_p(V^*_p) \) and the patent fee, \( P \). Condition (20b) again states the smooth-pasting condition. The solution to Eq. (19) has to take the form

\[
F_p(V) = \psi_e V^\beta \tag{21}
\]

where \( \beta > 0 \) and \( \psi_e > 0 \). This time, parameter \( \beta \) is the (positive) solution to quadratic equation

\[
(1/2)\sigma^2 \beta (\beta - 1) + \mu \beta - (r + \lambda) = 0. \tag{22}
\]

At this stage, if \( V \geq V^*_p \), two potential cases arise depending on whether \( V^*_p \) is greater or smaller than \( V^*_r \). From Eq. (14) we can see that \( F_D(V^*_r) = \psi_e(V^*_r)^\alpha \) if \( V^*_p < V^*_r \) (in which case the innovator obtains a development option only) and

\textsuperscript{15}Bosworth et al. (1996) report that the share of unpatented innovations is substantial. Indeed, only about half of product innovations are covered by patent applications while only about one third of process innovations are covered.
It may be optimal for the firm to choose secrecy instead of patenting. In such a case, the value of secrecy with delayed commercialization exceeds the value of the patent option. Denote the project value under secrecy by \( V_s \). It is easy to show that the project value at the date of market introduction is proportional to the development cost, though in contrast to Eq. (14) with patent protection, the factor of proportionality is given by \( \beta \), i.e. \( V_s = \beta D / (\beta - 1) \). Therefore, at the date of market introduction under secrecy, it holds that \( V_s - D > F_p(V) \). This amounts to saying that the value of the patent option falls short of the value of secrecy and can be rewritten as \( F_p(V) < D / (\beta - 1) \). It is indeed expected that this condition holds when the patent is narrow and its effective duration is short. For any given patent width, the economic value of the patent option directly measures the importance of the innovation. Thus, economically important innovations will typically be patented. One of the merits of the real options approach hence is that the decision to apply for a patent is endogenized instead of being treated as exogenous.

In order to analyze the optimal commercialization, it is useful to prove the following result:

**Lemma 1.** For any positive patent width \( \beta / \alpha > 1 \).

**Proof.** The only difference between Eq. (12) and Eq. (22) is the difference in the patent width, that is, \( \omega > 0 \). Then the result \( \beta > \alpha \) follows immediately from (17). QED

Stated verbally, \( \beta / \alpha \) is the ratio of the elasticities of the patent and the development options with respect to project value. **Lemma 1** states that the patent option is typically more elastic with respect to project value than is the development option. **Lemma 1** can be used to establish also that \( V_s < V_f \). This means that the threshold for market introduction with no patent falls short of the threshold value under patent protection, as our Proposition 1 actually suggested.

In general, the initial investments in research and the final investments on market introduction are separated, as Pigou (1934) pointed out in his *Industrial Fluctuations*: “There is no close connection in time between the discovery of these things and their exploitation.” However, the earlier research on real options has suggested a strong message that the threshold values of consecutive investments must decline over time. In our model, this would mean that \( V_s > V_f \). Intuitively, this case would arise from the facts that a priori the firm faces a sequence of future sunk costs and that the threshold value can be expected to be relatively high at the early stage of the program. Subsequently, when there are less sunk costs to be paid, the threshold value will be lower. One of the key findings of our paper, however, is that this condition need not hold. We can state the precise condition when \( V_s < V_f \). It reads as \( P / D < (\beta - \alpha) / (\alpha - 1) \). By **Lemma 1**,
\[ \beta > \alpha \]. The implication is that the successive real options will not be exercised at the same time:

**Proposition 2.** If the ratio of patent cost relative to the development cost is not too great, it is not an optimal industrial strategy to exercise the development option simultaneously with the patent option but rather to postpone the development.

For proof, we refer to Appendix A. As to the economics of Proposition 2, we observe that between optimal dates of patenting and market introduction, the patent ‘sleeps’. Previous research has explained ‘sleeping patents’ in a slightly different way. Dasgupta et al. (1982) consider R&D with exhaustible natural resources and find that an inventor with a patent chooses to wait until the stock of natural resources is sufficiently small and, accordingly, the price sufficiently high to justify the introduction of a new substitute product. In Gallini and Karp (1989), a monopolist-innovator may find it profitable to postpone introduction of new product until there are sufficiently many potential customers. In these studies, and in many others as well, patent protection is perfect and it is automatically and immediately awarded to the inventor upon the moment of discovery.

As one could intuitively anticipate, Proposition 2 suggests that the smaller are the patenting costs relative to the costs of commercializing the innovation, the earlier is the date at which the patent is applied for. This finding is supported by the empirical observations in Pakes (1986) that patents are often applied for at rather early stages during the R&D process. Moreover, the optimal timing of the patent depends on the discrepancy between \( \beta \) and \( \alpha \), which in turn is a function of the patent width. Clearly, the difference \( \beta - \alpha \) is strictly increasing in patent width because \( \beta \) is unrelated to \( w \). Therefore, broad patents make it more likely that the options will be exercised successively, not simultaneously. For valuing the patent option (21), it remains to find out the value of constant \( \psi_i \). By inserting 
\[ \psi_i(V_i^*) = \beta P^r(\beta - \alpha) \]
into Eq. (20a') in Appendix A, we obtain 
\[ \psi_i = \alpha P^r[(\beta - \alpha)(V_i^*)] \]
a result which will be needed below.

6. Research option: initial investment

Dixit and Pindyck (1995) proposed that the major economic purpose of R&D investments is to create options for valuable market opportunities. In the current section, we formalize their insight in a simple way and extend it by introducing an idea of an information revelation through commitment. We analyze the problem of the optimal exercising of the option to undertake the research program. We have suggested above that without the initial investment, the option typically loses its value. The opportunity is open for the innovator for some time only. If he does not commit himself to research, another firm might do it, or he might never find out the true value of the original idea. In terms of our model, it is research effort
which eliminates the veil that otherwise prevents the idea from reaching a state which would justify further action.

Recall that the value of research option, \( F_R \), will evolve under Eq. (1) as
\[
F_R = \psi V^\beta
\]
subject to the constraint that there will be a barrier \( V=b \), \( F'_R(V)=0 \).
This implies that \( \psi=0 \), making the research option worthless, a property which we suggested above. By committing itself to an investment, however, the firm learns with a positive probability, \( p(R) \), that the project is valuable and evolves according to Eq. (1) with no barrier. Therefore, with probability \( p(R) \) the idea qualifies for potential submission of a patent application. With probability \( 1-p(R) \), the firm, however, learns that the project is worthless. Thus, the initial investment is regarded as information revealing creating a patent option with probability \( p(R)>0 \) with \( p'(R)>0, \ p''(R)<0 \). That convexity can be assumed follows from diminishing returns to research effort. As an equilibrium condition the associated value-matching condition now reads as \( R=pF_p(V^*_p) \). This just states the equality between the cost of the project and the expected return and can be used to solve for the minimal threshold value of the project, \( V^*_p \), which justifies the initial research investment. Using Eq. (21), we obtain \( F_p(V^*_p)=[\alpha P/((\beta-\alpha))(V^*_p/V^*_r)]^\alpha \).

The incentive to apply for the patent early arises where \( V^*_p/V^*_r > 1 \) while the incentive to postpone the patent decision arises when \( V^*_p/V^*_r < 1 \). Inserting the expression of \( F_p(V^*_p) \) in \( R=pF_p(V^*_p) \), we then obtain
\[
\left(\frac{V^*_p}{V^*_r}\right)^\alpha = \left(\frac{R}{p}\right) \left(\frac{\beta-\alpha}{\alpha}\right) \left(\frac{1}{p}\right).
\]

Let us use the ratio of the research cost to success probability, \( R/p \), to measure the difficulty of the project. Then, because broad patents tend to raise the ratio of the elasticities (Lemma 1) \( \partial[(\beta-\alpha)/\alpha]/\partial \omega > 0 \), we can state a result which follows from Eq. (23).

**Proposition 3.** The incentive to postpone the patent application beyond the research phase arises if the project is not ‘too’ difficult, if the patent width is small, and if the patent is relatively costly.

The message of this result is that patents tend to be applied for in the case of difficult projects and they tend to be applied for relatively early. The cost of patenting plays an important role in the above result. It can be noticed that this cost may be different for small than for large firms. While the direct cost may be relevant for a small firm it is much less relevant for a large firm. However, it is plausible that unlike small firms, large firms more often have the economic resources and incentives more often to sue those infringing the patent. It is unlikely that capital markets would provide such resources for small firms, given potential informational asymmetries. Large firms thus can be predicted to be more
willing to bear such costs than small firms. If anything, these findings seem to provide testable hypotheses for econometric work.

7. Concluding remarks

The theory of innovations has recognized their sequential nature. In light of this, we have considered the commitment to an R&D project as a commitment to sequential investments where instead of an immediate cash flow the initial investment only leads to future options. After each decision, the regime (stochastic process) faced by the firm may be quite different from what it was before. Such a view has radical implications for the optimal research strategy. The current paper has shown that a patent may have two quite different effects on the speed of technical progress. First, a patent raises the ex ante present value of the rents of each potential project, thereby also enhancing the incentives for early market introduction. This effect may result in more projects being undertaken. Second, and in contrast to the first effect, a patent creates an option to delay market introduction of new products, thereby slowing down technical progress. It is the latter mechanism that has not received attention in the previous literature but has been the object of study in the current paper.

We have modeled a sequential innovation process in which each decision creates an option to continue or to terminate the ongoing research project. By exploring the effects of the patent system, it was possible to arrive at the conclusion that it cannot be taken for granted that patenting unambiguously speeds up technical progress. This result seems somewhat disturbing. However, what it really amounts to is a formalization of the intuitive idea that patenting provides more time for the development phase at reduced risk.

The model leads to hypotheses which could be tested empirically. For example, it predicts that innovators that leave innovations unpatented can be expected to use shorter lead times in industrial competition and earlier commercialization than innovators that acquire patents. In the latter case, one expects fast patenting if innovation has been ‘difficult’. A strong test would thus be provided by analyzing a sample of firms classified according to those which have and which have not filed patent application. Inter-country differences in patenting would be expected to show up in the average speed of patenting. There may also be inter-firm differences. The cost of patenting may be heavier for small than for large firms and large firms may have more economic resources and stronger incentives to sue those infringing the patent, etc.

Our findings also raise normative issues in the context of optimal technology policy. The role of patents in delaying the introduction of new products, not to mention the problem of unused patents, points to a potential social cost in terms of lost consumer surplus. We are not yet living in a Brave New World a’la Aldous Huxley (1977), where “The Invention Office is stuffed with the plans of labour-
saving processes... And why don’t we put them into execution? For the sake of labourers: it would be cruelty to afflict them with excessive leisure... Besides, we have our stability to think of. We don’t want to change. Every change is a menace of stability. That’s another reason why we’re so chary of applying new inventions.” Not to make too hasty conclusions, we do know that commitments to costly research are based on private cost-benefit evaluations and cannot be expected to be socially motivated. It may be that the delays caused by patents in development of products provide corrections for some distortions. On the other hand, to highlight the complexity of the normative issues, different firms may have different efficiencies in producing innovations and in developing commercialized products. It is thus not excluded that there may also be a social cost from delayed market introduction by the original innovator.

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Appendix A

Proof of Proposition 2

Suppose that \( V^*_{\tau} > V^*_{\tau'} \). Then \( F_D(V^*_{\tau}) = V^*_{\tau} - D \) and the boundary conditions (20a) and (20b) can be rewritten as

\[
F_p(V^*_{\tau}) = V^*_{\tau} - D - P. \tag{20a'}
\]

\[
F'_p(V^*_{\tau}) = 1. \tag{20b'}
\]

By Eq. (21) it holds that \( F_p(V^*_{\tau}) = \psi V^*_{\tau} \mu \), and using Eq. (20a’) and Eq. (20b’),
one can solve for \( V^* \) as \( V^*_t = \beta(D + P)/(\beta - 1) \). Now, comparing this expression to (14) we see that the case \( V^*_r > V^*_t \) arises if and only if the relative cost ratio satisfies \( \beta(P + D)/(\beta - 1) > \alpha D/(\alpha - 1) \). Rearranging yields \( P(D > (\beta - \alpha)/[\beta(\alpha - 1)] \), which is contrary to what it should be. Therefore, we know that in the case where the cost of commercializing is small relative to the cost of patenting the product, it pays to commit oneself both to the cost of the patent and the development cost simultaneously.

In contrast to the above case, it is necessary for \( V^*_r < V^*_t \) to hold that the option value of market introduction at time \( \tau^* \) satisfy \( F_{\tau^*}(V^*) = \psi_\tau(V^*)^\alpha \). Inserting this in Eq. (20a) and Eq. (20b) gives new boundary conditions

\[
\psi_\tau(V^*)^\beta = \psi_\tau(V^*)^\alpha - P \quad (20a')
\]
\[
\beta \psi_\tau(V^*)^\beta - 1 = \alpha \psi_\tau(V^*)^\alpha - 1 \quad (20b')
\]

Combining these two equations we obtain \( \psi_\tau(V^*)^\alpha = \beta P/(\beta - \alpha) \) which states the relationship between the cost of patenting and the option value at the optimal date of patenting. After substituting the expression for \( \psi_\tau \) from Eq. (15) and some simple calculations utilizing Eq. (14), we obtain the patenting rule as \( V^*_r = V^*_t[\beta P(\alpha - 1)/(D(\beta - \alpha)]^{1/\alpha} \). Clearly then \( V^*_r < V^*_t \) if \( [\beta P(\alpha - 1)/(D(\beta - \alpha)]^{1/\alpha} < 1 \), i.e. \( P(D > (\beta - \alpha)/[\beta(\alpha - 1)] \). QED

References


